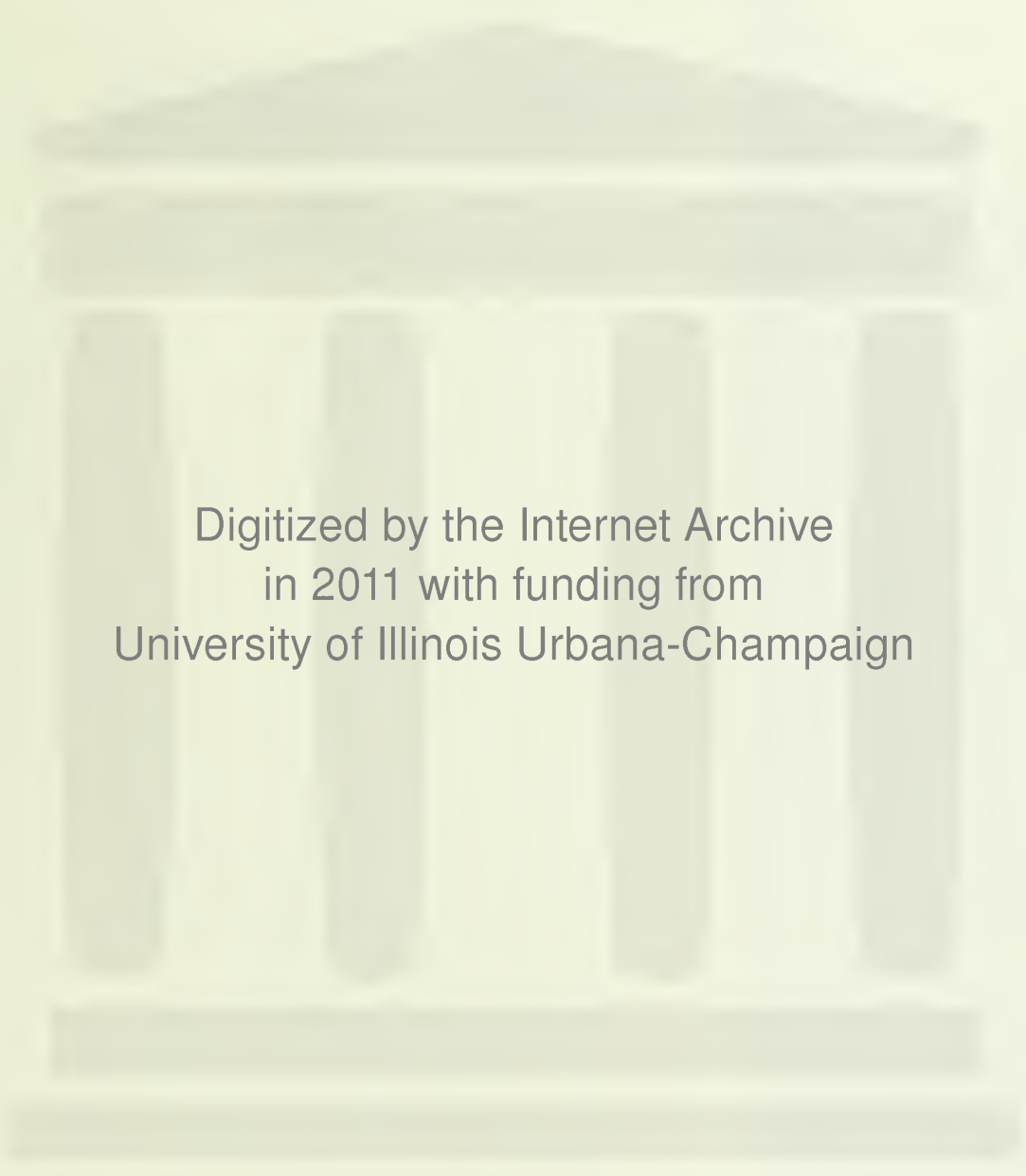


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Efficiency, Randomization, and Commitment in Government Borrowing

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Abstract

We study the problem of a government that wishes to share optimally the burden of deficit finance among heterogeneous agents with differential access to inside investment opportunities, when the government is not able to commit explicitly to honor its liabilities. In the presence of private information, it is Pareto efficient for the government to borrow in a way that amounts to non-linear taxation, and it must treat agents with access to the best investment opportunities preferentially to keep them in the bond market. The fact that some agents have access to other investments is a powerful force allowing the government to voluntarily make payments on its inherited debt, if the deficit is sufficiently large. In addition, with private information about access to assets, it is often desirable to randomize extraneously the return on the highest yielding government liabilities. The optimal government policy is shown to accord well with historical observations and provides insight into why randomization is not observed in most private contracts.

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1 Introduction

This paper considers the problem of a government with a deficit to finance which is faced with the following circumstances. First, the government cannot compel agents to hold its liabilities, so they must be induced to do so. Second, some agents have access to investments other than government liabilities, so the government must issue liabilities that are competitive with these investments. Third, there is private information on the part of agents about access to these investment opportunities. Finally, there are overlapping generations of agents who are not inter-generationally altruistic. Thus agents care only in a limited way about whether the government has honored its obligations in the past, or will do so in the (sufficiently distant) future. In the context of this environment we pose two questions.

- (1) What is the optimal way for the government to finance its deficit (from the perspective of steady state welfare)?
- (2) How does the government commit implicitly to honor its liabilities when explicit commitment is impossible?

We describe conditions under which optimal deficit finance involves the government issuing as many liabilities as there are agent types, and prevents them from being intermediated. These liabilities are issued in minimum denominations, and different liabilities bear different rates of return. In addition, some of the government's liabilities bear extraneously randomized returns, while others are riskless. Agents holding risky government debt diversify by holding some privately issued investments. When the government deficit is sufficiently large, it turns out that it is relatively easy for agents to induce the government to honor its debt obligations, even if the government does not care about the welfare of the (old) agents who hold its inherited debt. Interestingly, for implicit commitment to be possible, it is necessary that the government deficit be "large enough," in a sense to be made precise.

While there is obviously a large literature on optimal deficit finance

schemes and commitment, we appear to be the first to have posed questions (1) and (2) in this context. Bryant and Wallace (1984) considered the question of how to finance a given government deficit in an overlapping generations model with homogeneous generations. However, they allowed exogenous government commitment to repayment, full information, and agents had no access to any investments other than government liabilities. In this context, Bryant and Wallace showed that it is optimal for the government to issue its liabilities in (indivisible) large denominations, and to prevent them from being intermediated. Villamil (1988) extended the Bryant-Wallace analysis by incorporating heterogeneity within generations and allowing agents to be privately informed about their “type.” In addition to the features described by Bryant and Wallace, in her environment the optimal deficit finance scheme requires the government to issue as many liabilities as there are agent types, with different liabilities bearing different rates of return. However, as in Bryant-Wallace, Villamil allowed no investment opportunities other than government liabilities, and allowed for exogenous government commitment.

The Bryant-Wallace and Villamil analyses are successful efforts to explain why a government issues various kinds of liabilities with different return streams, and allows them to co-exist via legal restrictions on intermediation.¹ However, they do not explain why the government chooses to honor its obligations, nor do they consider the constraints imposed on the government by the necessity of competing with other investment opportunities. Tabellini (1991) examines an overlapping generations model with agent heterogeneity and private information in which the government can default on its inherited debt if a majority of voters approve such an action. He shows that there

¹Hicks (1935) gave the classic statement of the question: how does government issued fiat money co-exist with other (default free) government liabilities that dominate it in rate of return? Wallace (1983) suggested legal restrictions as an answer to this question. Bryant and Wallace (1984) and Villamil (1988) show that such legal restrictions are desirable when the government wants to behave as a price discriminating monopolist for reasons of optimal taxation. See Villamil (1992) for further discussion.

need not be defaults if there is sufficient inter-generational altruism. However, for Tabellini, implicit commitment requires inter-generational altruism, and he also does not allow agents access to assets not issued by the government. Also, neither Bryant-Wallace/Villamil nor Tabellini explain why a government might extraneously randomize the returns on its liabilities.

We believe that the extraneous randomization of returns on government liabilities, and the co-existence of government liabilities with other assets are important features of the appropriate answers to the questions we have posed. Extraneous randomization of the returns on government bonds is a surprisingly common feature of historical government borrowing schemes.² And in market economies government liabilities must compete with other assets. Moreover, it is often the case that some individuals (e.g., the wealthy) have access to investment opportunities that are not open to other agents. A government wishing to borrow from a broad spectrum of agents is constrained by the necessity of keeping agents with access to the best alternative investments in the market for its bonds. This leads to interesting issues of distribution as well as efficiency that are discussed in detail by Keynes (1940). We show that, under the conditions described in Section 5, the optimal way to keep agents in the bond market when they have access to good alternative investments is to offer them bonds with high expected, but extraneously randomized returns in large minimum denominations. Moreover, the necessity (or desirability) of keeping these agents in the bond market imposes considerable discipline on the government, preventing defaults under conditions that are discussed in Section 4.

Our vehicle for analyzing these issues is a stationary, two-period lived overlapping generations model in which a government with a utilitarian social welfare function must finance a fixed deficit of a given size. It does this by borrowing from two types of agents that are identical in all respects but one:

²This assertion is documented in some detail in Section 6.

different agents have different access to investment opportunities other than government bonds.³ We assume that agent type and investment activity are private information. If agents did not have differential investment opportunities the government would raise revenue from all types equally. When the deficit is sufficiently large, doing so drives agents with the best investment opportunities (say type 1 agents) out of the bond market. This, in turn, requires all revenue to be raised from type 2 agents, which a utilitarian government regards as undesirable. Thus the government raises as much revenue as it can from type 1 agents without driving them out of the market, and this requires that they be treated preferentially. When type is private information, preferential treatment of type 1 agents creates an adverse selection problem which optimal government policy must address.

Under conditions we describe, the government treats type 1 agents preferentially by designing an asset for them with a randomized return. In contrast, the asset designed for type 2 agents has a lower expected (but certain) return. Despite the fact that both agent types have identical preferences, endowments, and equal access to the government's assets, the access of type 1 agents to an outside alternative allows them to partially insure against the bad state of nature associated with the randomized return. Thus, type 1 agents have non-trivially diversified portfolios. Type 2 agents, having no access to the outside alternative, prefer the certain return. In addition, government liabilities are issued in minimum denominations, intermediation is prohibited, and there are as many types of government bonds (bearing different returns) as there are agent types. We show that this policy is constrained Pareto efficient if absolute risk aversion decreases at a rapid enough rate, because it is then the optimal way to keep type 1 agents in the bond market

³This captures situations where wealthier investors have access to investment opportunities not open to poorer investors, or where a government seeks to borrow both at home and abroad, and foreign investors have opportunities not open to domestic investors.

given the adverse selection problem.⁴ The potential desirability of extraneous randomization in environments with private information has, of course, been previously noted by Prescott and Townsend (1984) and Arnott and Stiglitz (1988).⁵ Our model differs from theirs in that randomization is desirable here only because agents have differential access to alternative (non-government) investment opportunities. This causes agents who are (otherwise) intrinsically identical to have indirect utility functions that differ in such a way that randomized allocations are Pareto superior to ones with no randomization.⁶

Finally, we believe our analysis sheds light on two other issues of general interest. The first is the question, raised by Arnott and Stiglitz (1988) among others, of why extraneous randomization does not arise as often as theory seems to suggest. Arnott and Stiglitz suggest six potential answers: (1) agents do not understand that randomization is optimal (perhaps because they are only boundedly rational); (2) randomized contracts may be costly to enforce; (3) secondary markets or insurance neutralize the effects of randomization; (4) agents view lotteries as unfair; (5) expected utility theory is deficient; or (6) individuals do not trust randomization mechanisms. As Section 6 indicates, governments have historically made heavy use of bonds with randomized returns—in a way that is consistent with our theory—so long as they could prevent bonds from being intermediated. This suggests

⁴The result that bonds with extraneously randomized returns are constrained Pareto efficient in markets with adverse selection can be interpreted as asserting the desirability of random taxation [cf., Stiglitz (1982)]. However, our focus is on government bond sales when participation cannot be compelled, so it is somewhat different from standard taxation analyses. In a taxation context, our model can be regarded as one in which only market activities can be taxed and high enough taxation drives some agents into non-market (or “underground”) activities. Thus, taxation must not only raise sufficient revenue, it must also be designed to prevent exit from market activities. The adverse selection problem arises in this setting if and only if voluntary participation is a binding constraint.

⁵Randomization may be desirable in the presence of private information due to the non-convexities it introduces. The possibility that randomization may be desirable in other contexts with non-convexities (for instance, indivisibilities) is discussed by Rogerson (1988) and Shell and Wright (1993).

⁶This insight is essentially the same as that in Benhabib, Rogerson, and Wright (1992).

that only (3) is persuasive in light of historical observation. Second, a large literature exists which argues that some subset of individuals is “liquidity constrained” (that is, they cannot borrow as much as they would like given market interest rates), and views this as evidence of a market failure. We show that an optimal government deficit finance scheme has the feature that some (but not all) agents perceive themselves to be liquidity constrained. This is not due to market failure, but is a consequence of the fact that the government optimally inhibits private borrowing and lending.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 considers non-stochastic planning problems from which Pareto efficient consumption allocations can be derived under three alternative sets of assumptions about the constraints faced by the planner. Section 4 examines when the government will voluntarily choose to honor its inherited debt, and Section 5 establishes conditions under which randomized allocations are desirable. In both Sections 3 and 5 we describe how the government can decentralize the optimal allocations. Section 6 discusses historical public debt policies and shows that they are consistent with the predictions of the model. Finally, Section 7 concludes.

2 The Model

Consider a discrete time economy populated by an infinite sequence of two-period lived, overlapping generations and an infinitely lived government. Each generation is identical in size and composition, containing a continuum of agents with unit mass. Within each generation there are two types of agents, indexed by $i = 1, 2$. Let θ_i denote the fraction of type i agents in each generation, with $\theta_i > 0$ and $\theta_1 + \theta_2 = 1$. In addition, there is a single consumption good at each date. All agents have endowment w_j of the good in period $j = 1, 2$ of their life, with $w_j \geq 0$.

Agent types are differentiated by their access to a storage technology.

Type 1 (and only type 1) agents have access to a constant returns to scale technology for storing the good, where one unit stored at time t returns $x \in (0, 1)$ units at time $t + 1$. Assume that each agent can store only his or her own good, that agent type is private information (ex-ante), and that the activity of storing the good (or the quantity stored) is unobservable.

All agents have identical preferences, representable by the additively separable utility function $u(c_1^i) + v(c_2^i)$, where $c_j^i \in \mathbb{R}_+$ denotes the consumption of a type i agent at age j . Assume that u and v are strictly increasing, strictly concave, and thrice continuously differentiable, and define $R(c) \equiv -\frac{v''(c)}{v'(c)}$ to be the coefficient of absolute risk aversion. Finally, let the government have an exogenously given real per capita expenditure level of $g > 0$ each period. Assume that agents derive no utility from this expenditure.⁷

The assumption that type 1 agents can store the good while type 2 agents cannot is meant to capture the problem facing a government which has a deficit to finance, and must borrow from a set of heterogeneous agents with differential access to alternative investment opportunities. The access of some agents to relatively high return investments limits the ability of the government to extract resources from them. In the model, the ability of type 1 agents to store the good gives them access to an asset not available to type 2 agents (the simplest form the problem can take). It proxies for several different scenarios. For example, wealthier agents might have access to investments not available to poorer agents.⁸ Alternatively, it could represent the situation of a government which seeks to borrow from foreign investors, who have investment options (bearing the gross rate of return x) not available to domestic investors. Finally, our model can be interpreted as an economy in which direct taxation is employed, but only “market activities” can be taxed and the differential access to the storage technology

⁷Or, government expenditure could affect utility in an additively separable way.

⁸We assume that all agents have identical endowments, but it is not difficult to let type i agents have an age j endowment of w_j^i with $w_1^1 > w_1^2$ [cf., Villamil (1988)].

proxies for different “non-market” opportunities.

For future reference, it will be useful to have a notation for the savings behavior of an agent who pays a lump-sum tax of τ_j at age j , and faces a certain gross rate of return on savings of r . Such an agent chooses a savings level, q , to maximize $u(w_1 - \tau_1 - q) + v(w_2 - \tau_2 + rq)$ subject to non-negativity constraints. The solution to this problem is given by the savings function $q \equiv f(w_1 - \tau_1, w_2 - \tau_2, r)$. Under our assumptions, and assuming interiority, $f_1 > 0 > f_2$. Also, we assume that

$$w_1 > f(w_1, w_2, x) > 0. \tag{a.1}$$

Finally, we define the indirect utility function V in the standard way:

$$V(w_1 - \tau_1, w_2 - \tau_2, r) \equiv u(w_1 - \tau_1 - f(\cdot)) + v(w_2 - \tau_2 + rf(\cdot)).$$

3 Non-random Pareto Efficient Allocations

This section describes the allocations which solve a utilitarian social welfare maximization problem under three alternative sets of assumptions about constraints faced by a social planner. It then considers how these allocations can be decentralized by a government which sells bonds competitively, but can impose legal restrictions on bond trades. Attention is restricted to non-random consumption allocations. Throughout, it is assumed that the government can commit to honor its future obligations. Whether the government would choose to do so when explicit commitment is impossible is considered in Section 4.

3.1 Full Information

As a benchmark, we begin by considering the problem of a social planner under full information. We assume that the planner knows each agent’s type, and can observe and (if desired) prohibit storage of the good. The planner’s

objective is to find a stationary allocation that maximizes an equally weighted sum of the agents' utilities subject to a resource feasibility constraint. Let k denote the amount of storage by a type 1 agent. The full information Pareto problem can be written as follows:

Problem 3.1. *For $i = 1, 2$, choose values c_1^i , c_2^i and k to maximize:*

$$\sum_{i=1}^2 \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to:

$$\sum_{i=1}^2 \theta_i (c_1^i + c_2^i) + \theta_1 k \leq w_1 + w_2 - g + \theta_1 xk. \quad (1)$$

At an interior optimum, the solution to this problem sets

$$u'(c_1^i) = v'(c_2^i), \quad (2)$$

for $i = 1, 2$, $c_j^1 = c_j^2$, for $j = 1, 2$, and $k = 0$. Notice that, from (1) and (2), $c_1^i = w_1 - f(w_1, w_2 - g, 1)$, and $c_2^i = w_2 - g + f(w_1, w_2 - g, 1)$. Thus the utility of agents under this allocation is given by $V(w_1, w_2 - g, 1)$.

Remark. The allocation given by the solution to Problem (3.1) is identical to that obtained by Bryant and Wallace (1984), and can be decentralized as they describe: The government can prohibit goods storage, sell bonds with a minimum real value of F and rate of return r , and prohibit agents from intermediating bonds. If F and r are chosen to satisfy $F = f(w_1, w_2 - g, 1)$ and $r = \frac{F-g}{F}$, each agent will voluntarily purchase bonds with a minimum real value of F [when $V(w_1, w_2 - g, 1) \geq V(w_1, w_2, 0)$]. This policy permits the government to raise enough revenue to cover its expenditure.

This [Bryant and Wallace] arrangement has the feature that the government sells indivisible, large denomination bonds. In addition, each agent saves more than he or she would prefer at the going rate of return in a market without restrictions on intermediation. Specifically, all individuals would

like to borrow against the future income from their investments (to consume more now), but are precluded from doing so by legal restrictions. Thus, all agents perceive themselves as liquidity constrained. This serves to emphasize that the existence of liquidity constraints need not be a signal of “market failure.” A government financing a deficit might choose to interfere with liquidity provision as part of an optimal deficit finance scheme.

3.2 Voluntary Participation

We now assume that the planner is subject to a voluntary participation constraint, or in other words, that the planner cannot prevent type 1 agents from autarchically storing the good or type 2 agents from consuming their endowments. This represents the situation of a government that must finance a deficit g by selling bonds, where the government is unable to compel bond purchases. Alternatively, we may view this as the situation of a government that cannot tax activities in an “underground” economy. We continue to assume that the government observes agents’ types directly.

The planner now solves the problem

Problem 3.2. *For $i = 1, 2$, choose c_1^i , c_2^i , and k to maximize:*

$$\sum_{i=1}^2 \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to: (1) and

$$u(c_1^1) + v(c_2^1) \geq V(w_1, w_2, x); \quad (3)$$

$$u(c_1^2) + v(c_2^2) \geq u(w_1) + v(w_2). \quad (4)$$

There are three possibilities regarding the solution to Problem (3.2).

Case 1: $V(w_1, w_2 - g, 1) \geq V(w_1, w_2, x)$. In this case (3) and (4) do not bind. This occurs, obviously, if g is sufficiently small, in which case the solution to Problem (3.2) is the same as the solution to Problem (3.1).

Case 2: $V(w_1, w_2 - g, 1) < u(w_1) + v(w_2)$. In this case the constraint set is empty. We abstract from this possibility, which occurs if g is too large.

Case 3: $V(w_1, w_2, x) > V(w_1, w_2 - g, 1) \geq u(w_1) + v(w_2)$. In this case (3) binds. This is the situation of interest to us so we focus exclusively on it. The solution satisfies (1) and (3) as equalities, (2), $c_j^1 > c_j^2$, for $j = 1, 2$, and $k = 0$. The solution to Problem (3.2), in this case, has the following feature. Due to its inability to compel agents to purchase its bonds, the government must offer type 1 agents relatively attractive terms to keep them in the bond market. They therefore attain a higher utility level than type 2 agents. However, since (2) holds, no inefficiencies result.⁹

Remark. The allocation given by the solution to Problem (3.2) can be decentralized by the following government policy. Bonds are sold to type i agents with a minimum real value of F^i and gross rate of return r^i . Then $F^i = w_1 - c_1^i$ and $r^i = \frac{c_2^i - w_2}{F^i}$ hold. Type 2 agents are prohibited from buying type 1 bonds, and intermediation is prohibited ex cathedra. Arguments following those of Bryant and Wallace (1984) establish that type i agents voluntarily purchase F^i units of bonds of type i . It is easy to verify that this permits the government to raise revenue equal to its expenditure.

This arrangement has all the features of Problem (3.1) except that the government issues many types of bonds bearing alternative rates of return. However, individuals' portfolios are not diversified.

3.3 Voluntary Participation and Private Information

We next consider the problem of a planner who wishes to choose non-stochastic Pareto efficient consumption allocations but cannot compel market participation, and in addition, cannot directly observe the type of any agent. Thus, the planner is subject to incentive compatibility constraints, as well as the other constraints specified previously.

⁹This corresponds to first degree price discrimination (or lump sum taxation).

The planner now solves the problem

Problem 3.3. For $i = 1, 2$, choose c_1^i, c_2^i and k to maximize

$$\sum_{i=1}^2 \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to: (1), (3), (4), and the self-selection constraints

$$u(c_1^1) + v(c_2^1) \geq u(c_1^2) + v(c_2^2); \quad (5)$$

$$u(c_1^2) + v(c_2^2) \geq u(c_1^1 + k) + v(c_2^1 - xk). \quad (6)$$

(5) imposes that type 1 agents weakly prefer (c_1^1, c_2^1) to (c_1^2, c_2^2) .¹⁰ (6) imposes incentive compatibility for type 2 agents, since a type 2 agent taking a type 1 allocation cannot mimic the storage of type 1 agents. He therefore consumes $c_1^1 + k$ when young and $c_2^1 - xk$ (i.e., c_2^1 less the proceeds of storage) when old.

Let $\tilde{c}_j^i$, $i, j = 1, 2$, and $\tilde{k}$ denote the solution to Problem (3.3). The solution can fall into one of two general categories.

Case 1: $V(w_1, w_2 - g, 1) \geq V(w_1, w_2, x)$. In this case the allocation from Problem (3.1) satisfies (3) through (6), since $c_j^1 = c_j^2$, for $j = 1, 2$.

Case 2: $V(w_1, w_2, x) > V(w_1, w_2 - g, 1)$. In this case the allocation from Problem (3.2) clearly is not incentive compatible, since $c_j^1 > c_j^2$, for $j = 1, 2$, and $k = 0$. In particular, since there is no goods storage and type 1 agents are “better treated” than type 2 agents, all type 2 agents will claim to be of type 1. We now focus on this case. It is clear that if (3) holds with equality, then (5) will be satisfied. Hence (1), (3), and (6) are the binding constraints in Problem (3.3). Moreover, the constraint set will be non-empty if, for instance, $V(w_1, w_2 - \frac{g}{\theta_2}, 1) \geq u(w_1) + v(w_2)$ holds.

We now characterize the solution to Problem (3.3).

¹⁰Formally, (5) should be written as $u(c_1^1) + v(c_2^1) \geq V(c_1^2, c_2^2, x)$. However, since $u'(c_1^2) = v'(c_2^2) > xv'(c_2^2)$ holds (see below), $V(c_1^2, c_2^2, x) = u(c_1^2) + v(c_2^2)$.

Proposition 1. *The solution to Problem (3.3) satisfies (1), (3), and (6) at equality, and has $u'(\tilde{c}_1^1) = xv'(\tilde{c}_2^1)$, $u'(\tilde{c}_1^2) = v'(\tilde{c}_2^2)$, and $f(w_1, w_2, x) > \tilde{k} > 0$.*

Proof. See Appendix A.

Remark 1. The solution to Problem (3.3) has at least two interesting features. First, as Appendix A shows, goods storage occurs. This is necessary to give type 1 agents a utility level of $V(w_1, w_2, x)$ without having type 2 agents mimic their bond purchases. Second, since $u'(\tilde{c}_1^1) = xv'(\tilde{c}_2^1)$, type 1 agents are “on their savings functions” with respect to storage of the good. Both of these features reflect inefficiencies due to the necessity of treating type 1 agents preferentially in the presence of private information.

Remark 2. To decentralize the solution to Problem (3.3) the government issues two types of bonds, and prevents intermediation. Agents who buy type 1 bonds with gross return x can buy only type 1 bonds, and are permitted to purchase *at most* $\bar{F}$ units (in real terms). Agents who purchase type 2 bonds with gross return r^2 must purchase at least F^2 units (in real terms). The government chooses F^2 and r^2 to satisfy $F^2 = w_1 - c_1^2$ and $r^2 = \frac{c_2^2 - w_2}{F^2}$, and $\bar{F}$ to satisfy

$$\bar{F} = f(w_1, w_2, x) - \tilde{k} > 0. \quad (7)$$

Then type 1 agents are “on their savings functions.” Type 2 agents optimally purchase F^2 units of type 2 bonds, and the government raises revenue with a per capita value of g .

Remark 3. This arrangement has the feature that type 1 agents hold diversified portfolios, since they store goods and hold government bonds. Moreover, type 1 agents are “on their savings functions,” and hence do not perceive themselves to be “liquidity constrained.” Type 2 agents do, since $u'(c_1^2) = v'(c_2^2) > r^2 v'(c_2^2)$. Thus, this context indicates that government imposed legal restrictions can lead some, but not all, agents to perceive themselves to be liquidity constrained. This does not signal a “market failure,”

as it is simply a consequence of actions taken by the government to finance its deficit in the most efficient manner possible (given its constraints).

4 Commitment

The previous section analyzed the optimal policy for a government engaged in financing a deficit when (a) it cannot compel bond purchases, and hence must be concerned about keeping agents in the bond market, (b) there is private information about agent type, and (c) the government can commit in advance to honor its obligations in the future. This section considers whether this policy is feasible when (c) is relaxed; that is, when the government cannot explicitly bind itself to honor its future obligations. More specifically, we examine whether it is possible to find an equilibrium where schemes that decentralize the solution to Problem (3.3) have the property that the government chooses to honor its inherited liabilities at each date (even though it is not bound to do so). To make our results as strong as possible, we assume that (i) the government cares only about young agents at each date,¹¹ and (ii) only a “one-time default” is considered. That is, agents are assumed to attach probability zero to a future default, even if a default has already occurred. Obviously, both assumptions work against the feasibility of commitment. We derive conditions under which the government voluntarily pays off on all its inherited liabilities at each date nonetheless.

The equilibrium that (potentially) allows implicit commitment operates as follows. Suppose the government pursues a scheme that decentralizes the solution to Problem (3.3), and suppose further that a default occurs at date t . If a default occurs at t , young type 1 agents are assumed to withdraw from the bond market at t and in all subsequent periods. When (3) binds this is individually rational for each type 1 agent, since these agents are

¹¹This is the strongest assumption we can make against the ability to commit, since obviously old agents will want inherited liabilities to be honored.

indifferent between lending to the government and investing autarchically. When a complete default occurs on all government bonds (which is the best situation for young agents at t), the government saves the resources which it promised to pay to old agents at t . This resource savings is $\theta_1(\tilde{c}_2^1 - x\tilde{k}) + \theta_2\tilde{c}_2^2 - w_2$,¹² which can be transferred to the current young. However, young type 1 agents withdraw from the bond market, so the resources young type 1 agents contribute to financing the deficit are lost to the government at t . This contribution is $\theta_1(w_1 - \tilde{c}_1^1 - \tilde{k})$; that is, young type 1 agents contribute their endowment less the sum of their young period consumption and their investment in storage. Thus the net resource gain to the government (at t) in the event of a default is

$$D \equiv \theta_1(\tilde{c}_2^1 - x\tilde{k}) + \theta_2\tilde{c}_2^2 - w_2 - \theta_1(w_1 - \tilde{c}_1^1 - \tilde{k}). \quad (8)$$

Using (1) in (8) gives an alternative expression for this net resource gain:

$$D \equiv \theta_2(w_1 - \tilde{c}_1^2) - g. \quad (8')$$

For all dates after t , type 1 agents are absent from the bond market (autarky). In these periods the government maximizes a weighted sum of young agent utilities, subject to the appropriate resource constraint, ignoring the possibility of further defaults. Since the utility of (autarchic) type 1 agents is $V(w_1, w_2, x)$, the government (after t) solves the problem

Problem 4.1. *Choose c_1^2 and c_2^2 to maximize*

$$\theta_1 V(w_1, w_2, x) + \theta_2 [u(c_1^2) + v(c_2^2)]$$

subject to (4), and the resource constraint with type 1 agents absent:

$$\theta_2(w_1 + w_2) - g \geq \theta_2(c_1^2 + c_2^2). \quad (9)$$

¹²This expression is the difference between the consumption promised to old agents ($\theta_1\tilde{c}_2^1 + \theta_2\tilde{c}_2^2$) and the sum of their endowments (w_2) plus their return on storage ($\theta_1 x\tilde{k}$).

Let (c_1^*, c_2^*) denote the solution to Problem (4.1).¹³ It is straightforward to show that $u(c_1^*) + v(c_2^*) = V(w_1, w_2 - g/\theta_2, 1)$. Since it is feasible in Problem (3.3) to set $c_1^2 = c_1^*$, $c_2^2 = c_2^*$ and $k = f(w_1, w_2, x)$, clearly $u(\tilde{c}_1^2) + v(\tilde{c}_2^2) > V(w_1, w_2 - g/\theta_2, 1) \equiv V(w_1 - g/\theta_2, w_2, 1)$.

We now consider the problem of the government at date t . This differs from Problem (4.1) in two respects. First, at date t the government has realized resource saving D . Second, it may choose an allocation different from the one which will be chosen at subsequent dates. In any event, this utilitarian government will seek to maximize a weighted sum of young agent utilities, subject to the appropriate resource constraints. Since young type 1 agents are autarkic at t , the government solves the problem:

Problem 4.2. Choose c_1^2 and c_2^2 to maximize

$$\theta_1 V(w_1, w_2, x) + \theta_2 [u(c_1^2) + v(c_2^2)]$$

subject to (4) and the resource constraints

$$\theta_2(w_1 - c_1^2) + D \geq g. \quad (10)$$

$$c_2^2 \leq w_2 + w_1 - c_1^* - g/\theta_2. \quad (11)$$

Equation (10) asserts that at t the resources provided by type 2 agents alone—plus the resource gain due to default—must finance both young type 2 consumption and government expenditures. Equation (11) states that the old age consumption of these agents cannot exceed their endowment, plus the resources provided by young agents at $t + 1$.

From (9) and (11) it is evident that $c_2^2 = c_2^*$ will hold in the solution to Problem (4.2). Therefore, the solution to this problem yields young type 2 agents at t a utility level of $u[w_1 - (g - D)/\theta_2] + u(c_2^*)$. Obviously this cannot exceed $V(w_1 - g/\theta_2, w_2, 1)$ if $D \leq 0$ holds. Thus, $D \leq 0$ is a sufficient

¹³If the solution exists. A solution will exist if the constraint set is non-empty: that is, if $\theta_2(w_1 + w_2) \geq g$ and $V(w_1, w_2 - g/\theta_2, 1) \geq u(w_1) + v(w_2)$.

condition for a default at t to reduce the utility of young type 2 agents; or equivalently, to reduce the value of the (time t) government's objective function. Thus, there is no incentive for a default to occur if

$$D \equiv \theta_2(w_1 - \tilde{c}_1^2) - g \leq 0 \quad (12)$$

holds.¹⁴ (12) is equivalent to

$$\tilde{c}_1^2 \geq w_1 - g/\theta_2, \quad (12')$$

so there is no default (the government honors its commitments) if the consumption of young type 2 agents is sufficiently large in Problem (3.3).

Remark 1. If (12') holds, no agent favors a default at any date under the assumptions stated (default makes the old worse off, and does not raise the utility of the young). Thus (12') is sufficient to guarantee unanimous opposition to government default.

Remark 2. The government never chooses to default if (12') holds because its reliance on type 1 agents imposes considerable discipline. The fact that type 1 agents are indifferent between investing in government bonds and investing elsewhere means that they are quite willing to “walk away” whenever there is any question about the government's commitment to honor its obligations. This observation is reminiscent of Alexander Hamilton's notion that government debt held by the wealthiest citizens (presumably those with access to other investments) creates a commonality of interest between these citizens and the government.¹⁵ For commitment to be possible, type 1 agents must be indifferent between holding government debt and other assets. This indifference obtains if and only if the government deficit is sufficiently large.¹⁶ Interestingly, Hamilton also felt that a large debt was conducive to a commonality of interests between a democratic government and its debtholders.

¹⁴(12) is sufficient for there to be no default, but it is not necessary.

¹⁵For a discussion of Hamilton's views on this point see Ferguson (1961).

¹⁶Specifically, (3) must bind in Problem (3.3).

Remark 3. Our focus on the desirability of a one-time default (under the assumption of no further defaults) can be given the following interpretation. Each (dated) government announces that it will not default. Then we examine the incentives for a deviation from this strategy by the government at time t , given the announced strategies of other (dated) governments. (12') gives a sufficient condition under which no government will wish to deviate from this strategy.¹⁷

5 Pareto Efficient Randomization

Having derived conditions under which the government will always choose to honor its obligations (if they are non-stochastic in nature), we now state conditions under which extrinsic randomization by the government can be Pareto improving. We first do so under the assumption that the government can explicitly commit to fulfilling its obligations. We then show that if (12') holds, commitment is always feasible in the presence of randomized government obligations. We begin by introducing some notation, and then consider a constrained social planning problem that permits extraneous uncertainty. Our objective is only to show that some extrinsic randomization is desirable, so we proceed as follows. We assume that the planner chooses, for $i = 1, 2$, deterministic values c_1^i for young consumption, and values $c_2^i(s)$ for old consumption that may depend on an extraneous state s . For simplicity we let $s \in \{1, 2\}$, and let $p \in (0, 1)$ be the exogenous probability (which is the same in all periods) that $s = 1$.¹⁸ We assume that realizations of s are independently and identically distributed across agents, and that s is realized at the

¹⁷This formulation of how policies are selected in the absence of an ability to commit is similar to that of Cooley and Smith (1989).

¹⁸Our description treats p as exogenous, but clearly randomization can be no less desirable on welfare grounds if the government is free to choose p .

beginning of old age.¹⁹

We now consider the planning problem from which constrained, Pareto efficient (possibly stochastic) consumption allocations are chosen. To simplify notation, we will sometimes write $E_s h(c_2^i(s)) \equiv p h(c_2^i(1)) + (1-p) h(c_2^i(2))$, where $h(\cdot)$ is an arbitrary function, and E denotes the expectation operator.

The stochastic Pareto problem can be written as follows.

Problem 5.1. *For $i = 1, 2$ and $s = 1, 2$, choose c_1^i , $c_2^i(s)$, and k to maximize*

$$\sum_{i=1}^2 \theta_i [u(c_1^i) + E_s v(c_2^i(s))]$$

subject to:

$$\sum_{i=1}^2 \theta_i [c_1^i + E_s c_2^i(s)] + \theta_1 k \leq w_1 + w_2 - g + \theta_1 x k; \quad (13)$$

$$u(c_1^1) + E_s v(c_2^1(s)) \geq V(w_1, w_2, x); \quad (14)$$

$$u(c_1^2) + E_s v(c_2^2(s)) \geq u(c_1^1 + k) + E_s v(c_2^1(s) - xk); \quad (15)$$

$$u(c_1^1) + E_s v(c_2^1(s)) \geq u(c_1^2) + E_s v(c_2^2(s)); \quad (16)$$

$$u(c_1^2) + E_s v(c_2^2(s)) \geq u(w_1) + v(w_2). \quad (17)$$

(13) is the resource feasibility constraint, which reflects the fact that there is no aggregate randomness. (15) and (16) are self-selection constraints, and (14) and (17) are voluntary participation constraints.

Clearly the solution to Problem (5.1) coincides with the solution to Problem (3.1) unless (14) is binding. When (14) binds, so does (15), as in the previous section. In this case, (16) cannot bind, and we restrict attention to the case in which (17) does not bind. Thus, for the remainder of the section,

¹⁹This captures features of several historical randomization devices employed in government borrowing [cf., Section 6] where governments confronted individuals with random returns on some bonds while they faced little or no (as here) randomness with respect to total interest obligations on these bonds.

constraints (13) through (15) bind. It is easy to verify in this case that the solution to Problem (5.1) has $c_2^2(1) = c_2^2(2)$, so that only (or at most) type 1 agents face extrinsic uncertainty. This is consistent with the historical observation that governments with large deficits have made use of bonds involving randomization with attractive return distributions that are sold to agents with good alternative investment opportunities [cf., Section 6]. Also, it is easy to check that the solution to Problem (5.1) has $u'(c_1^2) = v'(c_2^2)$. Finally, arguments identical to those in Appendix A can be used to establish that $k > 0$ holds, and that

$$u'(c_1^1) = xE_s v'(c_2^1(s)). \quad (18)$$

Thus, type 1 agents are “on their savings functions,” as before.

5.1 Welfare Improving Randomization

We now state a sufficient condition for $c_2^1(1) \neq c_2^1(2)$ to hold, so that type 1 agents face extraneous uncertainty.

Proposition 2. *Suppose that $-\frac{(1-x)v''(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} > -\frac{v''(\tilde{c}_2^1)}{v'(\tilde{c}_2^1)}$ holds, where $\tilde{c}_1^1$, $\tilde{c}_2^1$, and $\tilde{k}$ are solutions to Problem (3.3). Then $c_2^1(1) \neq c_2^1(2)$.*

Proof. See Appendix B.

In the remainder of this section, we provide necessary and sufficient conditions for the inequality in Proposition 2 to hold. We then interpret our result, and find that when the elasticity of absolute risk aversion with respect to old age consumption is sufficiently large, extrinsic randomization is welfare improving. The inequality in Proposition 2 can be rewritten

$$\frac{R(\tilde{c}_2^1 - x\tilde{k})(1-x)v'(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} > R(\tilde{c}_2^1). \quad (19)$$

Note that $\frac{v'(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} < \frac{1}{1-x}$, so a necessary condition for (19) to hold is decreasing absolute risk aversion (with respect to old age consumption). We now derive a sufficient condition for (19) to hold.

Define the function G by

$$G(\tilde{c}_1^1, \tilde{c}_2^1, k) \equiv \frac{R(\tilde{c}_2^1 - xk)(1-x)v'(\tilde{c}_2^1 - xk)}{v'(\tilde{c}_2^1 - xk) - u'(\tilde{c}_1^1 + k)} - R(\tilde{c}_2^1). \quad (20)$$

Since $(\tilde{c}_1^1, \tilde{c}_2^1)$ satisfies $u'(\tilde{c}_1^1) = xv'(\tilde{c}_2^1)$, it follows that $G(\tilde{c}_1^1, \tilde{c}_2^1, 0) = 0$. Moreover, $u(\tilde{c}_1^1) + v(\tilde{c}_2^1) = V(w_1, w_2, x)$ holds, so that $(\tilde{c}_1^1, \tilde{c}_2^1)$ is completely determined and independent of k . Thus, G is effectively a function of k alone, and if $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$ for all $k > 0$, $G(\tilde{c}_1^1, \tilde{c}_2^1, \tilde{k}) > 0$ will hold. This, of course, is exactly (19).

Straightforward differentiation of (20) establishes that $G_3 > 0$ iff

$$\begin{aligned} -\frac{xR'(\tilde{c}_2^1 - xk)}{R(\tilde{c}_2^1 - xk)} &> -\left\{\frac{u'(\tilde{c}_1^1 + k)}{v'(\tilde{c}_2^1 - xk) - u'(\tilde{c}_1^1 + k)}\right\} \times \\ &\quad \left\{\frac{u''(\tilde{c}_1^1 + k)}{u'(\tilde{c}_1^1 + k)} - xR(\tilde{c}_2^1 - xk)\right\}. \end{aligned} \quad (21)$$

Since $xv'(\tilde{c}_2^1 - xk) \geq u'(\tilde{c}_1^1 + k)$ for all $k \geq 0$, a sufficient condition for $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$, for all $k \geq 0$, is

$$-\left\{\frac{(1-x)R'(\tilde{c}_2^1 - xk)}{R(\tilde{c}_2^1 - xk)}\right\} \geq xR(\tilde{c}_2^1 - xk) - \frac{u''(\tilde{c}_1^1 + k)}{u'(\tilde{c}_1^1 + k)}. \quad (22)$$

An alternative statement of (22) is obtained by multiplying both sides by $(\tilde{c}_2^1 - xk)$ to get

$$\begin{aligned} -\left\{\frac{(1-x)R'(\tilde{c}_2^1 - xk)(\tilde{c}_2^1 - xk)}{R(\tilde{c}_2^1 - xk)}\right\} &\geq xR(\tilde{c}_2^1 - xk)(\tilde{c}_2^1 - xk) - \\ &\quad \left\{\left(\frac{\tilde{c}_2^1 - xk}{\tilde{c}_1^1 + k}\right) \frac{u''(\tilde{c}_1^1 + k)(\tilde{c}_1^1 + k)}{u'(\tilde{c}_1^1 + k)}\right\}. \end{aligned} \quad (22')$$

(22') provides the result. It asserts that $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$ for all $k \geq 0$ if the elasticity of absolute risk aversion [with respect to old age consumption, $\frac{R'(\cdot)(\cdot)}{R(\cdot)}$], is sufficiently large. Note that $R'(\cdot) < 0$ if the utility function exhibits everywhere strictly decreasing absolute risk aversion.²⁰

²⁰Decreasing absolute risk aversion implies that in a choice between a safe and a risky asset, the risky asset is a normal good. This is a common assumption about preferences.

5.2 An Example

We now consider the special case in which $u(c_1) = \phi \frac{c_1^{1-\rho}}{1-\rho}$ and $v(c_2) = \frac{c_2^{1-\rho}}{1-\rho}$, with $\phi \geq x$ and $\rho > 0$. Then $-\frac{cR'(c)}{R(c)} \equiv 1$ for all c . In addition, $u'(\tilde{c}_1^1) = xv'(\tilde{c}_2^1)$ implies that $\tilde{c}_2^1 = \tilde{c}_1^1(\frac{x}{\phi})^{1/\rho} \leq \tilde{c}_1^1$, and consequently, that $\tilde{c}_2^1 - xk \leq (\tilde{c}_1^1 + k)(\frac{x}{\phi})^{1/\rho}$ for all $k \geq 0$. In this case (22') reduces to $1 - x \geq x\rho + \rho \frac{(\tilde{c}_2^1 - xk)}{(\tilde{c}_1^1 + k)}$ for all $k \geq 0$, which of course holds if $1 - x \geq x\rho + \rho(\frac{x}{\phi})^{1/\rho}$. Thus $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$ for all $k \geq 0$ holds if ρ is sufficiently small. This implies that the inequality in Proposition 2 holds, and that randomization is desirable on welfare grounds.

5.3 Decentralizing the Optimal Stochastic Allocation

We must slightly augment our notation from Section 2 to describe how to decentralize the optimal stochastic allocation. Consider the savings problem of a young agent who faces a random lump-sum tax of $\tau_2(s)$ when old, $s = 1, 2$, where the probability that $s = 1$ is p , and who faces a deterministic gross rate of return r . This agent's problem is to choose a savings level q to maximize $u(w_1 - \tau_1 - q) + pv(w_2 - \tau_2(1) + rq) + (1-p)v(w_2 - \tau_2(2) + rq)$. The solution to the problem is a savings function $q \equiv \tilde{f}(w_1 - \tau_1, w_2 - \tau_2(1), w_2 - \tau_2(2), r; p)$.

The optimal random consumption allocation can be supported by the following policy. The government sells two types of bonds, and prohibits intermediation. The bonds sold to type 2 agents are sold in a minimum denomination of F and bear a deterministic return r . The government chooses F and r to satisfy $c_1^2 = w_1 - F$ and $c_2^2 = w_2 + rF$. The bonds sold to type 1 agents are sold only in the indivisible amount $\hat{F}$, and bear a gross return $\hat{r}(1)$ with probability p , and $\hat{r}(2)$ with probability $1 - p$. The government chooses $\hat{F}$, $\hat{r}(1)$, and $\hat{r}(2)$ to satisfy

$$c_1^1 = w_1 - \hat{F} - \tilde{f}[w_1 - \hat{F}, w_2 + \hat{r}(1)\hat{F}, w_2 + \hat{r}(2)\hat{F}, x; p]; \quad (23)$$

$$c_2^1(1) = w_2 + \hat{r}(1)\hat{F} + x\tilde{f}(\cdot); \quad (24)$$

$$c_2^1(2) = w_2 + \hat{r}(2)\hat{F} + x\tilde{f}(\cdot). \quad (25)$$

This works since, by (14), type 1 agents are “on their savings functions.”

Bonds are sold in many indivisible denominations with alternative rates of return, and intermediation is restricted. There is individual portfolio diversification by type 1 agents, who store the good and hold government bonds bearing random returns. Moreover, these agents do not perceive liquidity constraints, while type 2 agents do. Finally, the government simultaneously issues bonds with randomized returns and bonds with certain returns.

5.4 Commitment

The preceding analysis assumed that the government could commit to honor its future debt obligations. If the government is not able to make such an explicit commitment, it is still possible that the government would choose to honor its inherited liabilities. In particular, when explicit commitment by the government is ruled out, it is possible to repeat the analysis of Section 4 for schemes that decentralize the solution to Problem (5.1). As in Section 4, a sufficient condition for the government to voluntarily make full payments on its inherited liabilities is

$$\hat{c}_1^2 \geq w_1 - g/\theta_2, \quad (26)$$

where $\hat{c}_1^2$ is the optimal choice of c_1^2 in Problem (5.1). It is straightforward to show that $\hat{c}_1^2 > \tilde{c}_1^2$ if the solution to Problem (5.1) involves non-trivial randomization.²¹ Thus whenever (12') holds, (26) holds also.

6 Some Evidence

We have considered the problem of a government with a deficit to finance that (a) cannot compel participation in the bond market by all agents from

²¹This follows from the fact that the solution to Problem (5.1) has $u(\hat{c}_1^2) + v(\hat{c}_2^2) \geq u(\tilde{c}_1^2) + v(\tilde{c}_2^2)$ (with strict inequality if the solutions to Problems (3.3) and (5.1) are different) and $u'(\hat{c}_1^2) = v'(\hat{c}_2^2)$. Thus $\hat{c}_j^2 \geq \tilde{c}_j^2$, $j = 1, 2$ holds, and the inequality is strict if there is randomization in the solution to Problem (5.1).

whom it wishes to borrow, and (b) cannot directly observe the characteristics of agents who purchase its liabilities. In addition, (c) some agents have access to investment opportunities other than government liabilities, and (d) agents have differential access to these investments. Under these conditions—and when the government can limit “intermediation” of its liabilities—we have demonstrated that the extraneous randomization of returns on government liabilities can easily be desirable from a welfare perspective. Moreover, the model makes the following predictions:

- (i) Extraneous randomization of returns on liabilities is desirable only when the government’s deficit is fairly large [(3) binds in Problem (5.1)].
- (ii) When extraneous randomization is employed, it occurs on bonds issued in large minimum denomination yielding relatively high expected returns.
- (iii) Extraneous randomization is observed only if the government can limit intermediation of its liabilities.

In this section we demonstrate some historical conditions where (a)-(d) were satisfied. During the periods we describe, governments made heavy use of liabilities with extraneously randomized returns, when their revenue needs were large. The use of such instruments ended when “intermediation” became sufficiently efficient. The liabilities we describe were issued in large denominations and offered relatively high expected returns. Thus the predictions of the model seem to be substantiated by historical observation.

6.1 Some Observations

During the 17th and 18th centuries, the British and French governments were frequently at war. During these periods of large deficits, both governments relied heavily on debt instruments with extraneously randomized returns. These instruments fell into three general categories.

1. *“Lottery-loans” or lottery-bonds.*²² Bond holders purchased a “ticket.”

²²The term “lottery-loans” is used by Jennings and Trout (1982).

The ticket entitled them to a minimum “prize” plus the chance at augmented earnings. For instance in the English lottery-loan of 1694, 100,000 tickets were sold at a price of 10 pounds each.²³ “Each ticket guaranteed a minimum return ... [of] one pound annually for sixteen years, an effective interest rate of 6 percent,” [cf., Jennings and Trout (1982), p. 28]. Prizes made the effective (mean) interest rate paid by the government 11.25 percent, which Jennings and Trout (1982, p. 29) indicate was high by current standards. In some lottery-loans the prizes were additional payments of bonds.

2. *Life annuities.* The purchaser paid the government a stipulated principal and named a nominee. The annuitant received interest as long as the nominee was alive. This made the return random to the holder of the obligation, while the government faced little or no payment randomness—it benefited from the ability to sell a large number of life annuities. The annuitant could name anyone as nominee, thus arranging a random return not contingent on his own life. The French government offered life annuities where multiple nominees could be listed, and the expected return offered was a decreasing function of the number of nominees listed.

3. *Tontines.* Devised by Lorenzo Tonti circa 1650, tontines had many features of life annuities, but in this case a fixed payment by the government was divided among debtholders whose nominees were alive. Thus the government faced little randomness in its payments if it had a large set of subscribers. The lottery aspects of government tontine sales (from the point of view of purchasers) were well recognized by both Tonti and the governments that employed tontines [cf., Jennings and Trout (1982), pp. 6, 26]. Tontines were sold in large minimum denominations and offered high expected returns [Weir (1989)]. The standard price for a tontine share in France was 300 livre—when “at the time of the French Revolution, many workers were

²³This is of the same order of magnitude as per capita income at the time. This substantiates the model’s prediction that these instruments would be issued in large denominations.

earning one or two livres a day” [cf., Jennings and Trout (1982), p. 2]. The price of one share in some 18th century English tontines was as high as 100 pounds.

Thus a large array of government liabilities with extraneously randomized returns was in use. These liabilities were sold in relatively large minimum denominations, and offered high expected returns. In addition to being used in England and France, tontines were employed at the same time in Denmark, Germany, and the Netherlands. Alexander Hamilton proposed the use of a tontine in the U.S. [cf., Jennings, Swanson, and Trout (1988)], and his 1790 proposal for restructuring the U.S. public debt involved the use of several instruments offering explicitly randomized returns [cf., Calomiris (1992)]. In England and France in particular these kinds of debt instruments were used intensively. In Britain the lottery “attracted more subscribers than any other form of loan” in the 1760s [cf., Dickson (1967), p. 54], and Weir (1989, p. 109) argues that “in Britain they [lotteries] appear to have been the rule rather than the exception.” In 1789 the largest component of French debt consisted of life annuities [cf., Weir and Velde (1992), p. 12.] And, after 1699, “lotteries became a standard source of public credit” in France [cf., Jennings and Trout (1982), p. 30].

The importance of the inability to compel bond market participation in the use of these kinds of debt instruments is illustrated by Dickson (1967) and Jennings and Trout (1982), who discuss the offerings of instruments that were under-subscribed.²⁴ The relevance of private information is considered by Dickson (1967, p. 78), who argued (in the English case) that “the Exchequer ... was never certain who its creditors were at any one time.” Finally, it is apparent that these liabilities were used primarily in periods of large deficits, since they were mainly issued in wartime.

²⁴A further illustration of the inability to compel participation is that “the [French] government even made them [its debt instruments] available to residents of nations at war with France,” [cf., Jennings and Trout (1982), p. 52].

For welfare improvements to result from extraneous randomization in government borrowing, the government must be able to limit agents from “intermediating” or “sharing” its liabilities. Agents did attempt to do exactly this. That governments sought to inhibit such intermediation is indicated by the fact that the English Parliament made sharing of lottery-loan tickets illegal in 1743-4, [cf., Dickson (1967), P. 507].²⁵ The French government also made some attempts to prevent life-annuities and tontines from being intermediated. “Some authors claim that France preferred inalienable life-contingent debt because there was no resale market...” [cf., Weir (1989), p. 111]. And a large component—more than 25 percent—of life annuity purchasers in France named themselves or a close relative as nominee. When this was the case the prospects for intermediation were very limited [cf., Weir and Velde (1992), pp. 32-3].²⁶

These observations are consistent with the predictions of our analysis. An alternative explanation for them is that governments were simply attempting to exploit a taste for gambling by running analogs to modern state lotteries. We reject this explanation for two reasons. First, if participants in the schemes were “risk-preferers,” then the relevant debt instruments should have borne no higher expected returns than alternative instruments with deterministic returns. Dickson (1967), Jennings and Trout (1982), Weir (1989), and Weir and Velde (1989) indicate that the instruments we describe bore high expected returns, and in fact appear to have been a surprisingly expensive way to borrow. Second, unlike modern state lotteries, “lottery-tickets” in our episodes were generally sold in quite large minimum denominations. As

²⁵It is also the case that a 1697 act of Parliament forbade brokers to deal in any government securities without the permission of the treasury [cf., Dickson (1967), p. 493].

²⁶An interesting intermediation of French tontines occurred in Geneva in the 1760s. Syndicates compiled a list of nominees of young girls who had survived smallpox and came from families with a history of longevity. They bought tontines naming these girls as nominees and sold shares in the syndicate. After this kind of intermediation became sufficiently widespread, the use of tontines by the French government was limited substantially.

would therefore be expected, Dickson (1967, p. 302) reports that in England these instruments were held by the middle and upper classes. In contrast, Clotfelter and Cook (1990) report that participants in modern state lotteries are primarily from lower socio-economic classes. Thus, the explanation we propose seems a reasonable one for the observed behavior of the British and French governments of the 17th and 18th centuries.

7 Conclusions

We have described an environment in which a government must finance a fixed deficit of a given size. When some agents have access to investment opportunities other than government bonds, government borrowing is constrained by the desirability of keeping these agents in the bond market. However, treating some agents preferentially creates an adverse selection problem. The optimal solution to these two problems involves price discrimination by the government, and may involve the simultaneous use of bonds with random and non-random returns.²⁷ Interestingly, agents with the best outside investment opportunities purchase bonds with random returns, and extraneous randomization of bond returns is observed only when the government's revenue needs are sufficiently large. These two features accord well with the historical observations cited in Section 6. And, when the government's revenue needs are large enough, the necessity of keeping all agents in the bond market turns out to be a powerful factor in allowing equilibria where the government voluntarily honors all of its inherited debt.

Finally, since the bond policy we describe is a form of price discrimination, the existence of secondary markets or insurance would render the government

²⁷This borrowing mechanism can also be interpreted as one in which inflation is random and both indexed and non-indexed government bonds co-exist, or as one where there is a hierarchy of claims against the government, and bonds bearing high expected returns are subject to some risk of partial default. Thus the model can confront a number of ways in which modern governments borrow using bonds with random returns.

unable to implement it. That is, implementation of the constrained Pareto efficient bond policy requires the government to impose legal restrictions that prohibit the intermediation of bonds with randomized returns. It is interesting to note that poorly developed financial and insurance markets are common in many high inflation countries that choose to monetize their deficits. For simplicity, we assume an *ex cathedra* prohibition against the intermediation of assets. However, Bencivenga and Smith (1992) study the optimal degree of financial repression in a developing economy faced with a sustained deficit that must be monetized. They find that a government with a deficit (that is either unwilling or unable to decrease spending or increase explicit taxes) may be required by simple feasibility to engage in financial repression to support its monetization program. Such repression is much more difficult in more developed countries and may be one reason why “lottery bonds” have not been observed more recently.

Another reason is that the existence of government liabilities issued in indivisible denominations can be a substantial source of indeterminacies when allocations are decentralized. Cooley and Smith (1993) establish the potential for the indeterminacy of perfect foresight equilibria in the Bryant-Wallace (1984) economy under their optimal deficit finance scheme. And Smith (1989) shows that minimum denomination restrictions can be a source of sunspot equilibria along the lines of Shell (1977), Azariadis (1981), and Cass and Shell (1983). The question of the optimal government policy subject to minimal constraints on the determinacy of equilibrium in this context is an interesting topic for further investigation.

8 Appendix A: Proof of Proposition 1

We first restate Problem (3.3) with only the binding constraints displayed.

Problem 3.3.' For $i = 1, 2$, choose c_1^i , c_2^i , and k to maximize

$$\sum_{i=1}^2 \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to

$$\sum_{i=1}^2 \theta_i (c_1^i + c_2^i) + \theta_1 k \leq w_1 + w_2 - g + \theta_1 x k \quad (A.1)$$

$$u(c_1^1) + v(c_2^1) \geq V(w_1, w_2, x) \quad (A.2)$$

$$u(c_1^2) + v(c_2^2) \geq u(c_1^1 + k) + v(c_2^1 - xk). \quad (A.3)$$

Proof of Proposition 1. Let $\lambda_n \geq 0$ be the Lagrange multiplier associated with constraint (A.n), and observe the following.

At interior solutions for c_1^1 and c_2^1 , the relevant first order conditions are

$$u'(c_1^1)(\theta_1 + \lambda_2) - \lambda_3 u'(c_1^1 + k) = \theta_1 \lambda_1. \quad (A.4)$$

$$v'(c_2^1)(\theta_1 + \lambda_2) - \lambda_3 v'(c_2^1 - xk) = \theta_1 \lambda_1. \quad (A.5)$$

The first order condition for k is

$$\lambda_3 [v'(c_2^1 - xk)x - u'(c_1^1 + k)] - \lambda_1 \theta_1 (1 - x) \leq 0, \quad (A.6)$$

with equality if $k > 0$.

Finally, the first order conditions for c_1^2 and c_2^2 at an interior optimum are

$$u'(c_1^2)(\theta_2 + \lambda_3) = \theta_2 \lambda_1. \quad (A.7)$$

$$v'(c_2^2)(\theta_2 + \lambda_3) = \theta_2 \lambda_1. \quad (A.8)$$

Of course (A.7) and (A.8) imply that $u'(c_1^2) = v'(c_2^2)$.

Now multiply both sides of (A.5) by x , subtract the result from (A.4), and use (A.6) to eliminate $\theta_1 \lambda_1 (1 - x)$ to obtain

$$(\theta_1 + \lambda_2)[xv'(c_2^1) - u'(c_1^1)] \leq 0, \quad (A.9)$$

with equality if $k > 0$.

We now establish that $k > 0$. To do so we suppose, for the purpose of deriving a contradiction, that $k = 0$. Then (A.4) and (A.5) imply that $u'(c_1^1) = v'(c_2^1)$. Since (A.2) is binding, it follows that $c_j^1 > c_j^2$, for $j = 1, 2$. But then (A.3) is violated, giving the desired contradiction. Thus $k > 0$.

It remains to establish that $f(w_1, w_2, x) > k$ holds. To do so we note that (A.2) and (A.9) hold with equality. These two equations yield a unique solution for c_1^1 and c_2^1 ; namely $c_1^1 = w_1 - f(w_1, w_2, x)$ and $c_2^1 = w_2 + xf(w_1, w_2, x)$. Since (A.2) is an equality, Problem (3.3') reduces to maximizing $u(c_1^2) + v(c_2^2)$, subject to (A.1) – (A.3). From (A.1) at equality, $\theta_2(c_1^2 + c_2^2 - w_1 - w_2) = \theta_1[w_1 + w_2 - c_1^1 - c_2^1 - (1 - x)k] - g = \theta_1(1 - x)[f(w_1, w_2, x) - k] - g$. Clearly then, it is not optimal to set $k \geq f(w_1, w_2, x)$. This completes the proof.

9 Appendix B: Proof of Proposition 2

We begin by considering the following augmented version of Problem (5.1).

Problem 5.1': For $i = 1, 2$ and $s = 1, 2$, choose c_1^i , $c_2^i(s)$, and k to maximize

$$\sum_{i=1}^2 \theta_i \{u(c_1^i) + pv(c_2^i(1)) + (1 - p)v(c_2^i(2))\}$$

subject to: (13) through (15) and

$$u'(c_1^1) = xp v'(c_2^1(1)) + x(1 - p)v'(c_2^1(2)). \quad (B.1)$$

Since the solution to Problem (5.1) satisfies (B.1), imposition of this constraint does not alter the optimal choices for the social planner. Equations

(13) through (15), which hold as equalities, and (B.1) constitute four equations involving c_1^1 , $c_2^1(1)$, $c_2^1(2)$, k , c_1^2 , and c_2^2 [since $c_2^2(1) = c_2^2(2) = c_2^2$]. We now use (13), (15), and (B.1) to eliminate c_1^1 , $c_2^1(1)$, and k from Problem 5.1'.

First, let (B.1) define c_1^1 as a function of $c_2^1(1)$ and $c_2^1(2)$. In particular, define $c_1^1 \equiv \alpha(c_2^1(1), c_2^1(2))$. Clearly, $\tilde{c}_1^1 = \alpha(\tilde{c}_2^1, \tilde{c}_2^1)$ holds. In addition, differentiation of (B.1) yields

$$\alpha_1(c_2^1(1), c_2^1(2)) = px \frac{v''(c_2^1(1))}{u''(c_1^1)} > 0. \quad (B.2)$$

$$\alpha_2(c_2^1(1), c_2^1(2)) = (1-p)x \frac{v''(c_2^1(2))}{u''(c_1^1)} > 0. \quad (B.3)$$

Second, substitute $c_1^1 = \alpha(c_2^1(1), c_2^1(2))$ into (13) at equality. This gives k as a function of $c_2^1(1)$, $c_2^1(2)$, c_1^2 , and c_2^2 . Thus define $k \equiv \beta(c_2^1(1), c_2^1(2); c_1^2, c_2^2)$. Observe that $\tilde{k} = \beta(\tilde{c}_2^1, \tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)$ holds, and that differentiation of $\beta(\cdot)$ yields:

$$\beta_1 = -\left(\frac{\alpha_1 + p}{1-x}\right) < 0. \quad (B.4)$$

$$\beta_2 = -\left(\frac{\alpha_2 + 1-p}{1-x}\right) < 0. \quad (B.5)$$

Third, substitute $c_1^1 = \alpha(c_2^1(1), c_2^1(2))$ and $k = \beta(\cdot)$ into (15) at equality. This defines $c_2^1(2)$ as a function of $c_2^1(1)$, c_1^2 , and c_2^2 ; say $c_2^1(2) \equiv \gamma(c_2^1(1); c_1^2, c_2^2)$. As before $\tilde{c}_2^1 = \gamma(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)$. Moreover, differentiation of $\gamma(\cdot)$ yields

$$\gamma_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = -\frac{p}{1-p}. \quad (B.6)$$

Finally, define the function $\delta(\cdot)$ as follows

$$\delta(c_2^1(1); c_1^2, c_2^2) \equiv u(\alpha(c_2^1(1), \gamma(\cdot))) + pv(c_2^1(1)) + (1-p)v(\gamma(\cdot)). \quad (B.7)$$

Observe that $\delta(\cdot)$ is the left-hand-side of constraint (14), the (binding) voluntary participation constraint for type 1 agents. The function $\delta(\cdot)$ expresses the left-hand-side of this constraint solely as a function of $c_2^1(1)$ and c_j^2 , for

$j = 1, 2$. The strategy of the remainder of the proof is to show that $\delta(\cdot)$ is locally convex in $c_2^1(1)$, so local randomization relaxes the voluntary participation constraint on type 1 agents and consequently is welfare improving.

Now observe that Problem 5.1' reduces to the following:²⁸

Problem 5.1''. Choose c_1^2 , c_2^2 , and $c_2^1(1)$ to maximize

$$u(c_1^2) + v(c_2^2)$$

subject to:

$$\delta(c_2^1(1); c_1^2, c_2^2) \geq V(w_1, w_2, x). \quad (B.8)$$

If $c_2^1(1) = \tilde{c}_2^1$ at an optimum, then the solution to Problem 5.1'' coincides with the (non-stochastic) solution to Problem 3.3.

We now establish the following properties of $\delta(\cdot)$:

$$\delta_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = 0, \quad (B.9)$$

and if the inequality in Proposition 2 holds,

$$\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) > 0. \quad (B.10)$$

Then setting $c_2^1(1) \neq \tilde{c}_2^1$ (in some neighborhood of $\tilde{c}_2^1$) relaxes constraint (B.7) in Problem 5.1''. It follows that at an optimum, $c_2^1(1) \neq \tilde{c}_2^1$, and consequently $c_2^1(1) \neq c_2^1(2)$. Thus there will be extraneous randomization of the allocation received by type 1 agents.

It remains, then, to establish that (B.9) and (B.10) hold. For (B.9), straightforward differentiation of (B.7) gives

$$\delta_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = u'(\alpha(\cdot))[\alpha_1 + \alpha_2\gamma_1] + pv'(\tilde{c}_2^1) + (1-p)v'(\tilde{c}_2^1)\gamma_1. \quad (B.11)$$

Substitution of (B.2), (B.3), and (B.6) into (B.11) gives (B.9)

²⁸This follows since $u(c_1^1) + pv(c_2^1(1)) + (1-p)v(c_2^1(2)) = V(w_1, w_2, x)$ holds.

For (B.10), further differentiation yields

$$\begin{aligned} \delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) &= u''(\tilde{c}_1^1)[\alpha_1 + \alpha_2 \gamma_1]^2 + u'(\tilde{c}_1^1)[\alpha_{11} + \alpha_{12} \gamma_1 + \alpha_{21} \gamma_1 + \alpha_{22}(\gamma_1)^2 + \alpha_2 \gamma_{11}] \\ &\quad + p v''(\tilde{c}_2^1) + (1-p)v''(\tilde{c}_2^1)(\gamma_1)^2 + (1-p)v'(\tilde{c}_2^1)\gamma_{11}. \end{aligned} \quad (B.12)$$

It is straightforward but tedious to show that when evaluated at $(\tilde{c}_2^1, \tilde{c}_1^2, \tilde{c}_2^2)$

$$\begin{aligned} \alpha_{11} + \alpha_{12} \gamma_1 + \alpha_{21} \gamma_1 + \alpha_{22}(\gamma_1)^2 + \alpha_2 \gamma_{11} &= \\ \frac{1-p}{p} \alpha_1 \left\{ \gamma_{11} + \left[\frac{p}{(1-p)^2} \right] \frac{v'''(\tilde{c}_2^1)}{v''(\tilde{c}_2^1)} \right\}; \end{aligned} \quad (B.13)$$

$$\alpha_1 + \alpha_2 \gamma_1 = 0; \quad (B.14)$$

$$\begin{aligned} \gamma_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)(p + x \alpha_1) &= -\frac{p x \alpha_1 v'''(\tilde{c}_2^1)}{(1-p)^2 v''(\tilde{c}_2^1)} - \\ &\quad \frac{p^2 v''(\tilde{c}_2^1 - x \tilde{k})(1-x)}{(1-p)^2 [v'(\tilde{c}_2^1 - x \tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})]}. \end{aligned} \quad (B.15)$$

Substituting (B.13) through (B.15) into (B.12) gives

$$\begin{aligned} \delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) &= \frac{p v''(\tilde{c}_2^1)}{1-p} + \frac{x \alpha_1 v'(\tilde{c}_2^1) v'''(\tilde{c}_2^1)}{(1-p) v''(\tilde{c}_2^1)} + (1-p) v'(\tilde{c}_2^1) \gamma_{11} \left[1 + \frac{x \alpha_1}{p} \right] \\ &= \frac{p v''(\tilde{c}_2^1)}{1-p} - \frac{v'(\tilde{c}_2^1) p (1-x) v''(\tilde{c}_2^1 - x \tilde{k})}{(1-p) [v'(\tilde{c}_2^1 - x \tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})]}. \end{aligned} \quad (B.16)$$

Clearly $\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) > 0$ if the inequality in Proposition 2 holds. This completes the proof.

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